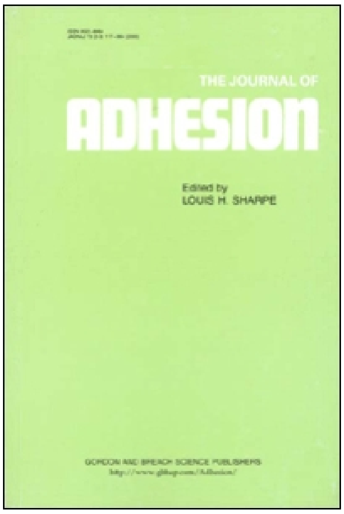


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# Fracture Mechanics of aV-peel Adhesion Test - Transition from a Bending Plate to a Stretching Membrane

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# Fracture Mechanics of a V-peel Adhesion Test – Transition from a Bending Plate to a Stretching Membrane

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A linear elastic solution is proposed for a "V-peel" adhesion test for a thin film adhered to a rigid substrate. The mechanical responses of a stiff plate-like coating under pure bending, a semi-flexible film under mixed bending and stretching, and a flexible membrane-like film under pure stretching are discussed. For delamination to occur, the mechanical energy release rate is shown to be $G = \chi(Fw_0/2bl)$ with $\chi$ a numerical constant varying from 3/2 for a plate-like disc to 3/4 for a thin flexible membrane.

*Keywords:* V-peel test; pull-off experiment; adhesion; delamination; thin film

## 1. INTRODUCTION

Advancement in microelectronics requires rigorous characterization of thin films in terms of their mechanical properties and adhesion strength. Axially symmetric blister tests using a uniform pressure *via* a fluid medium (pressurized blister test) [1–3], or a central load *via* a shaft (shaft-loaded blister test) [4–6], are documented in the literature. Several notorious shortcomings regarding these adhesion measurement methods are cited. The pressurized test, which requires a sophisticated experimental setup to monitor the simultaneous change in pressure and blister dimension, suffers from the soft compliance of the

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fluid medium and possible dissolved gases. The shafted-loaded blister test, on the other hand, offers a more attractive alternative, because the universal testing machine offers a better compliance for stress-strain measurements. One setback is due to the concentrated central point load and the associated large membrane stress in the vicinity of the blister center, which, in turn, leads to plastic yielding or even film puncture.

One possible remedy to reduce the stress concentration is to apply the external load in a line, *i.e.*, the "pull-off" experiment [7] or the V-peel test (Fig. 1). Two opposite ends of the film are adhered to the sides of a rectangular opening in the substrate, while the other two edges remain free. An external force is then applied *via* a horizontal bar attached to the shaft end, so that the load is evenly distributed along the film width. The elastic solution for a plate under pure bending is well known [8] though not widely adopted as an adhesion test. The V-peel test for a thin flexible coating under pure stretching was studied by Gent and Kaang [7]. The transition from pure bending to pure stretching as a result of film thickness and stiffness is largely ignored in the literature. In fact, most mechanical characterizations of thin films assume either plate-like or membrane-like behavior *a priori*. Without any direct measurement of the degree of rigidity (or flexibility), or the ratio of bending to stretching, erroneous conclusions regarding the elastic modulus and adhesion strength are inevitable.



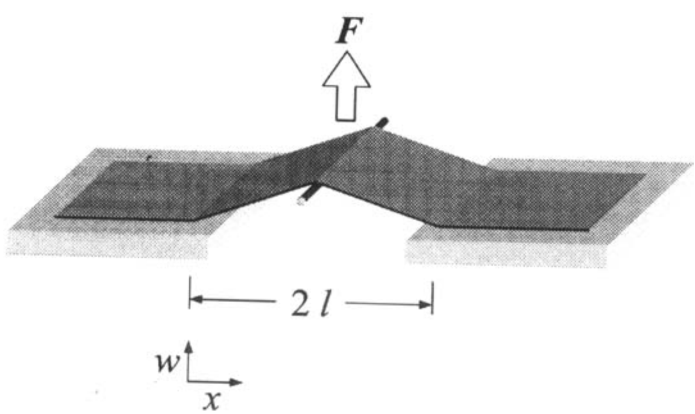


FIGURE 1 Schematic of a V-peel test showing the straight sided "V" geometry for a thin flexible film under pure stretching.

The V-peel test offers a way to determine the film rigidity, as well as other film properties.

In this paper, we will show how a parameter $\beta$, defined as the ratio of membrane stress to flexural rigidity, determines both the mechanical response of the film and the corresponding film profile. An exact elastic solution for the V-peel test and a mechanical energy release rate over the entire range of $\beta$ are calculated based on linear elasticity.

## 2. ELASTIC SOLUTION FOR A V-PEEL TEST

A strip of breadth, $b$, length, $2l$, thickness, $h$, elastic modulus, $E$ and Poisson's ratio, $\nu$, is adhered to a substrate as shown in Figure 1. An external load, $F$, is applied at the centerline *via* a horizontal wire. The film is, thus, elastically deformed into a V-shape under a mixed bending/stretching mode. The film profile is denoted by $w(x)$ with a central deflection, $w_0$. The bending moment, $M$, at any cross section of the strip is given by

$$M = (F/2)x - Nw + M_0 \tag{1}$$

where $x$ is the distance from one end, $N$ the membrane stress and $M_0 = M|_{x=l}$. Note that the membrane stress is present as a consequence of the applied load, *i.e.*, $N = 0$ when $F = 0$. In the presence of a residual stress, $N_{res}$, in the film, an extra term, $-N_{res}w$, is added to the right hand side of Eq. (1). For simplicity, we treat $N_{res} = 0$ in this paper. Since $M = -D\,(d^2w/dx^2)$ with the flexural rigidity $D = Eh^3/12(1-\nu^2)$, it can be shown by Eq. (1) that

$$\frac{d^2w}{dx^2} - \frac{N}{D}w = -\left(\frac{F}{2D}\right)x - \frac{M_0}{D} \tag{2}$$

For simplicity, a set of useful dimensionless parameters are defined as follows:

$$\xi = \frac{x}{l}, \quad W = \frac{w}{h}, \quad \theta = \frac{dW}{d\xi},$$

$$\beta = l\left(\frac{N}{D}\right)^{1/2} \quad \text{and} \quad \varphi = \frac{Fl^3}{2Dbh}$$

The parameter $\beta$ is defined to be the ratio of membrane stress to film rigidity such that $\beta \approx 0$ for pure bending and $\beta \to \infty$ for pure stretching. The solution to Eq. (2) must satisfy three boundary conditions, namely, (i) $\theta|_{\xi=1} = 0$, (ii) $\theta|_{\xi=0} = 0$ and (iii) $W|_{\xi=0} = 0$. Equation (2) is solved exactly yielding

$$W = \frac{\varphi}{\beta^3}\left\{ -\sinh(\beta\xi) + \left[\frac{\cosh\beta - 1}{\sinh\beta}\right][\cosh(\beta\xi) - 1] + \beta\xi \right\} \tag{3}$$

with a central deflection $W_0 = W|_{\xi=1}$ such that

$$W_0 = \frac{\varphi}{\beta^3}\left[ -\sinh\beta + \frac{(\cosh\beta - 1)^2}{\sinh\beta} + \beta \right] \tag{4}$$

Figure 2 shows the normalized profile $W(\xi)$. As the film becomes more flexible, the profile tends to a linear V-shape. The elastic strain on the film is given by

$$\frac{Nl}{E'h} = \int_0^l (\sec\theta - 1)dx \approx \frac{1}{2}\int_0^1 (dw/dx)^2 dx \tag{5}$$

where $E' = E/(1 - \nu^2)$ and $\sec\theta = 1 + \theta^2/2$ for small $\theta$. Substituting Eq. (3) into Eq. (5), the normalized applied load takes the form

$$\varphi = \frac{\beta^3}{[6f(\beta)]^{1/2}} \tag{6}$$

where

$$f(\beta) = \frac{2 + \cosh\beta}{2\cosh^2(\beta/2)} - \frac{2\sinh\beta}{\beta} + \frac{\cosh\beta\sinh 2\beta}{4\beta\cosh^2(\beta/2)} - \frac{3\tanh(\beta/2)}{2\beta} - \frac{2\cosh\beta\tanh(\beta/2)}{\beta} + \frac{\cosh 2\beta\tanh(\beta/2)}{2\beta} \tag{7}$$

Figure 3 shows a parametric plot of $\varphi(W_0)$ on a log–log scale with the parameter $\beta$ varying from 0.01 to 15. For a film of intermediate stiffness, $\varphi \propto (W_0)^n$ where $n$ is the gradient:

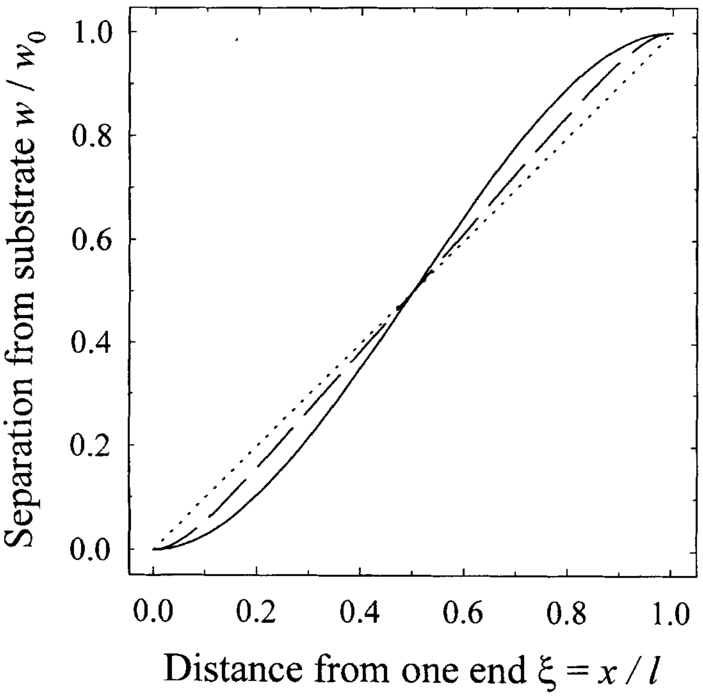


FIGURE 2 Normalized film geometry $w(x)/w_0$ as for $\beta = 1$ (solid line) and $\beta = 15$ (dashed line). The dotted line shows the linear V-shape geometry for pure stretching. Film profile is symmetric about $x = l$, or $\xi = 1$.

$$n = \frac{d(\log \varphi)}{d(\log W_0)} = \cosh^2(\beta/2)\left[\frac{\begin{array}{c}(6+2\beta^2)\cosh(3\beta/2)/\cosh(\beta/2)\\ -9\beta\sinh(3\beta/2)/\cosh(\beta/2)\\ +(3\beta+2\beta^3)\tanh(\beta/2)+(4\beta^2-6)\end{array}}{\begin{array}{c}(20\beta^2-21)+19\beta^2\cosh\beta\\ +(21+3\beta^2)\cosh(2\beta)\\ +(\beta^3-51\beta)\sinh\beta-(33\beta/2)\sinh(2\beta)\end{array}}\right] \tag{8}$$

Bending predominates when $\beta$ is small, $n = 1$, and $\varphi \propto W_0$; and stretching prevails at large $\beta$, $n = 3$, and $\varphi \propto (W_0)^3$. The transition occurs in an arbitrary range from $W_0 = 0.1$ to 20.

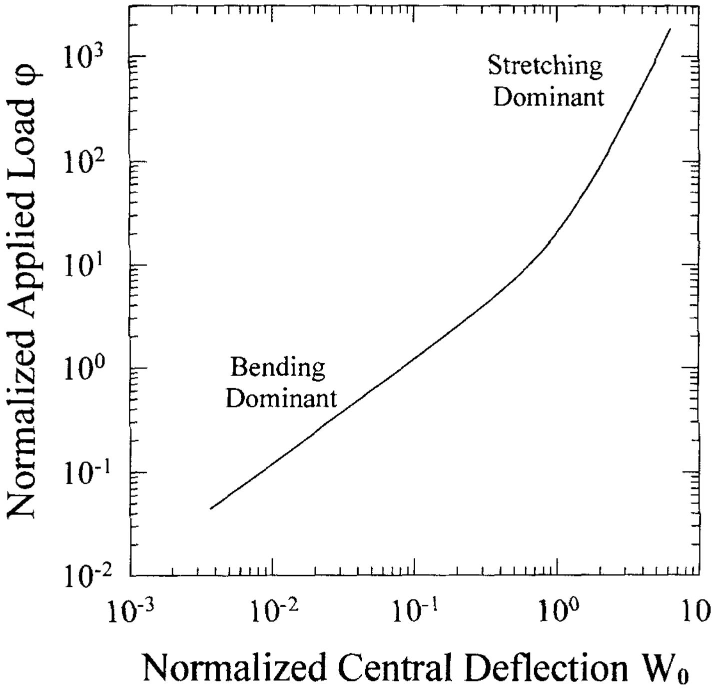


FIGURE 3 Normalized applied load $\varphi(=Fl^3/2Dh)$ as a function of normalized blister height, $W_0\ (=w_0/h)$. The "bending dominant" region has a slope of 1, and the "stretching dominant" region roughly 3. Transition occurs in the range from $W_0 = 0.1$ to 20.

In the limiting case of pure bending, Eqs. (3) and (4) reduce to

$$W = \frac{\varphi}{2}\left(\frac{\xi^2}{2} - \frac{\xi^3}{3}\right) \quad \text{or} \quad w = w_0\left[\frac{1}{2}\left(\frac{x}{l}\right)^2 - \frac{1}{3}\left(\frac{x}{l}\right)^3\right] \tag{9}$$

$$W_0 = \frac{\varphi}{12} \quad \text{or} \quad w_0 = \frac{Fl^3}{24bD} \tag{10}$$

respectively, which is the well-known classical pure bending solution [8]. The linear dependence of $W_0$ on $\varphi$ in Eq. (10) is the direct result of linear elasticity of bending. As for pure stretching, $f(\beta) \approx 1$ in Eq. (7) and $\varphi = \beta^3/6^{1/2}$ in Eq. (6), and the film profile becomes

$$W = (\varphi/\beta^2)\xi \quad \text{or} \quad w = w_0 x \tag{11}$$

$$W_0 = \varphi/\beta^2 \quad \text{or} \quad w_0 = \left(\frac{1-\nu^2}{Eh}\right)^{1/3}\left(\frac{F}{b}\right)^{1/3} l \tag{12}$$

respectively, *i.e.*, a linear V-shape. Eqs. (11) and (12) can also be derived by putting $D \to 0$ in Eq. (2) so that $(d^2w/dx^2) = 0$ and $M_0 = 0$. These solutions were first derived by Gent and Kaang, though the factor $(1-\nu^2)$ in Eq. (12) is replaced by $(1-\nu)$ in [7]. The difference is due to the fact that, in a thin membrane, the stretching stress is one-dimensional and the lateral contraction strain in the perpendicular axis is $(1-\nu)$. This factor of $(1-\nu)$ is preferred in the case of an ultra-thin film. Note that Eq. (12) can also be rewritten as $\varphi = 6W_0^3$, which represents a non-linear $\varphi(W_0)$ relation (*cf.* $\varphi = 12\ W_0$ in pure bending, Eq. (10)). It is interesting to remark that in this extreme situation $(\beta \to \infty)$, the boundary conditions (*i*) and (*ii*) are no longer matched for an exact linear V-shape profile. The thin film is now so flexible that virtually no bending stress exists even at the debonding front, resulting in a non-zero debonding angle. In a conventional peel test at a variable peel angle, which is a more general configuration compared with a V-peel test, it was shown earlier by Williams [9] that there always existed a local bending moment at the root.



## 3. FRACTURE MECHANICS OF A V-PEEL TEST

When the external load exceeds a threshold, delamination occurs. The mechanical energy release rate, $G$, for a V-peel test under a fixed load can be found by putting [10]

$$G = \frac{1}{2}\left(\frac{dU_c}{dl}\right)_F \tag{13}$$

where the complimentary energy is $U_c = \int w_0\ d(F(b)|_A$ with a fixed delamination area $A = 2bl$. The term $U_c$ can be normalized by putting $\Omega = U_c/(Fw_0/b)$ such that

$$\Omega = \frac{\int W_0 d\varphi}{\varphi W_0} = \frac{4\beta + 3\beta\cosh\beta - 7\sinh\beta}{8\cosh(\beta/2)[\beta\cosh(\beta/2) - 2\sinh(\beta/2)]} \tag{14}$$

It can be shown that $1/2 \leq \Omega \leq 3/4$. The lower bound of $\Omega = 1/2$ corresponds to a pure bending mode $(\beta \approx 0)$, while the upper bound of

$\Omega = 3/4$ corresponds to pure stretching ($\beta \to \infty$). Note that linear elasticity in pure bending requires a linear $F(w_0)$ relation; it follows that $U_c$ is identical to the elastic energy stored and is exactly $1/2\ Fw_0$ [8]. Therefore, Eq. (13) can be rewritten as $G = (\Omega F/2)\ (dw_0/dl)|_F$ where $F$ is fixed. Since $\varphi \propto l^3$ and $W_0 \propto \varphi^{1/n}$ (Fig. 3), a normalized mechanical energy release rate $\chi = G/(Fw_0/2bl)$ can be defined such that

$$\chi = \frac{3\Omega}{n}$$
$$= 3\left\{\frac{\begin{array}{c}(64\beta^3 - 90\beta)\cosh(\beta/2) + (9\beta + 14\beta^3)\cosh(3\beta/2)\\ +(81\beta + 6\beta^3)\cosh(5\beta/2) + (84 - 26\beta^2 + 10\beta^4)\sinh(\beta/2)\\ +(42 - 91\beta^2 + 6\beta^4)\sinh(3\beta/2) - (42 + 41\beta^2)\cosh(5\beta/2)\end{array}}{\begin{array}{c}16[\beta\cosh(\beta/2) - 2\sinh(\beta/2)]^2[36\beta\cosh(\beta/2)\\ +6\beta\cosh(3\beta/2) + (2\beta^2 - 21)\sinh(\beta/2)\\ -21\sinh(3\beta/2)]\end{array}}\right\} \tag{15}$$

Figure 4 shows $\chi(\beta)$. For a plate-like film, $\Omega = 1/2$ and $n = 1$, so $\chi_{\text{bend}} = 3/2$; and for a membrane-like film, $\Omega = 3/4$ and $n = 3$, so $\chi_{\text{stretch}} = 3/4$. This can be verified by substituting Eq. (10) into Eq. (13) and Eq. (12) into Eq. (13), respectively. The pure stretching result coincides with the solution given by Gent and Kaang [7]. The bending to stretching transition occurs in an arbitrary range from $\beta = 0.1$ to 20. To measure the adhesion strength of an interface, $G_c$, one has first to measure $n$ by determining the constitutive relation, Eq. (8) in a pre-delamination loading, followed by applying Eq. (15) in the course of delamination where the materials parameter, $G$, is a constant equal to $G_c$ (or $\chi = \chi_c$).

We can investigate the fracture stability of the V-peel test. It can easily be shown that $G \propto F^{(1+1/n)}\ l^{(3/n-1)}$, as $G \propto (Fw_0/l)$ and $w_0 \propto (Fl^3)^{1/n}$ (since $W_0 \propto \varphi^{1/n}$). In a quasi-static fracture process ($G = G_c$), $F \propto l^{(n-3)/(1+n)}$. In a pure bending plate with $n = 1$, there are two consequences: (*i*) $F \propto l^{-1}$, so that $F$ decreases as the crack grows and (*ii*) $G \propto l^2$, so that $G$ increases as the delamination area increases, *i.e.*, an unstable crack growth in a dead load configuration [11]. In a pure stretching membrane, $n = 3$, therefore, both $F$ and $G$ are independent

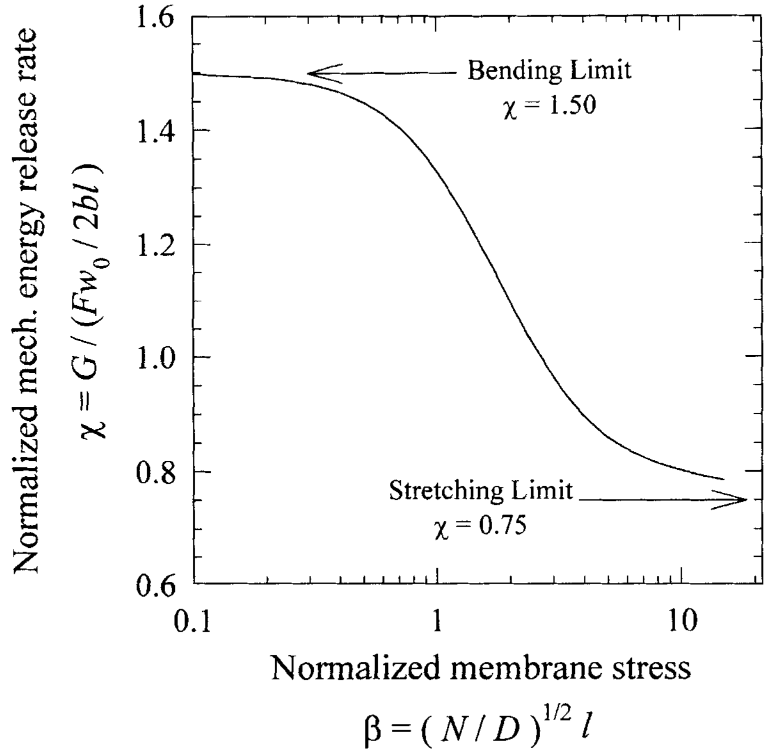


FIGURE 4 Normalized strain energy release rate $\chi[= G/(Fw_0/2l)]$ as a function of normalized membrane stress $\beta[= (N/D)^{1/2} l]$. Transitions occurs in the range from $\beta = 0.1$ to 20.



of $l$. As the crack propagates, $F$ remains constant until the film completely detaches from the substrate. For a film under mixed bending and stretching, crack stability lies between these two extremes and the crack is unstable.

## 4. DISCUSSION AND CONCLUSION

The V-peel test can be considered as the one-dimensional equivalent of the axially symmetric shaft-loaded blister test. Table I shows a comparison between the two tests in terms of $F(w_0)$ and $\chi = G/(Fw_0/A)$. In the case of pure bending, both configurations possess a similar linear

TABLE I Comparison between a V-peel test and a shaft-loaded blister test

| | *V-peel test* | *Shaft-loaded blister test* |
|---|---|---|
| Dimension | 1-D | 2-D |
| Delamination area, $A$ | $2bl$ | $\pi a^2$ |
| $F(w_0)$ – pure bending | $F = (l^3/24\ Db)\ w_0$ | $F = (4\pi\ E'h^3/3a^2)w_0$ |
| $F(w_0)$ – pure stretching | $F = (E'hb/l^3)w_0^3$ | $F = (\pi Eh/4a^2)w_0^3$ |
| $\chi$ – pure bending | 3/2 | 1/2 |
| $\chi$ – pure stretching | 3/4 | 1/4 |

$F(w_0)$; and in stretching, a similar cubic relation. The ratio of $(\chi_{bend}/\chi_{stretch})$ in both tests is identically equal to 2. The V-peel formulation can be also applied to the axially symmetric blister resulting in a similar bending-to-stretching transition, which will be discussed in a sequel paper [12].

In a thin film under pure stretching, the tensile membrane stress as a result of the centerline force can be compared with that of the central point load. When conducting a V-peel test experimentally, the horizontal bar has a finite radius, $R$, rather than an ideal dimensionless thin wire. The same is true for the shaft-loaded blister test, where the shaft tip is replaced by a steel ball of a finite radius. For simplicity, one can assume that the contact region between the film and the cylinder (or ball) experiences a uniform pressure and, thus, a uniform membrane stress. If the radii of the cylinder and the ball are the same, and the delamination areas, $A$, are identical (*i.e.*, $bl = \pi a^2$), then, at the same applied load, the ratio of the V-peel to the shaft-loaded blister height will be at (1/3) and the ratio of membrane stresses roughly $(R/b)$.

We have shown how the membrane stress ($\beta$) and the blister height ($W_0$) determine the degree of film rigidity, and the mechanical behavior with or without delamination. A plate-like response requires a thick film (large $h$), a stiff material (large $E$), a small span of strip (small $l$) and a small applied load (small $F$), while the opposites are true for a membrane-like behaviour. We have also shown how the mechanical energy release rate changes as the physical properties of the film varies. A plate-like behavior can be presumed as long as $\beta$ falls short of the transition range $0.1 \leq \beta \leq 20$, and a membrane-like behavior beyond the range. The results presented in this paper are useful in designing specimen dimensions and loading configuration for testing fragile coatings on microelectronic parts, and in deducing the materials parameters such as elastic modulus and adhesion strength.


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